B. Tech - Sem - 4

P P SAVANI UNIVERSITY

Fourth Semester of B. Tech. Examination

May 2019

SESH2022 Numerical and Statistical Analysis

13.05.2019, Monday **Time: 09:00 a.m. To 11:30 a.m.** **Maximum Marks: 60**

Instructions:

1. The question paper comprises of two sections.
2. Section I and II must be attempted in separate answer sheets.
3. Make suitable assumptions and draw neat figures wherever required.
4. Use of scientific calculator is allowed.

SECTION – I

Q - 1 Do as directed. (Any Five) **[05]**

(i) Can we apply Simpson's $\frac{1}{3}$ rule to find integration from 3 to 18 for following data?

x	3	6	9	12	15	18
$f(x)$	22	29	31	20	4	0

State your answer with appropriate reason.

(ii) Write nth iteration formula of Euler's method for solving ordinary differential equation?

(iii) Given an example of transcendental equation.

(iv) If exact value is 4 and approximate value is 3.6 then find percentage error.

(v) Find the interval in which the equation $x^4 - x - 10 = 0$ has real root.

(vi) Check whether the function $f(z) = z$ is entire function or not?

(vii) Determine the residue of the function $f(z) = \frac{1-e^{2z}}{z^4}$ at $z = 0$.

Q - 2 (a) Evaluate $\int Re(z^2)\,dz$ over C, where C is the boundary of the square with vertices $0, i, 1+i, 1$ in the clock wise direction. **[05]**

Q - 2 (b) Determine an analytic function whose real part is $e^x(x\cos 2y - y\sin 2y)$. **[05]**

OR

Q - 2 (a) Determine $F(1), F(5), F'(i), F''(-i)$ if $F(\alpha) = \oint \frac{3z^2-2z+1}{z-\alpha}\,dz$ over C, where C is an ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. **[05]**

Q - 2 (b) *i*) Check whether function $f(z) = z^{\frac{5}{2}}$ is analytic or not. **[2.5]**

ii) Find the image of semi-infinite strip $x > 0,\ 0 < y < 2$ when $w = iz + 1$. Sketch the strip and its image. **[2.5]**

Q - 3 (a) Find cubic spline valid on [3,4] for following data. Assume $M_0 = 30$ and $M_3 = 408$ **[05]**

x	1	2	3	4
$f(x)$	3	40	189	576

Q - 3 (b) *i*) The velocity of a train which starts from rest is given by the following table, the time being reckoned in minutes from the start and speed in km/h **[2.5]**

Time	3	6	9	12	15	18
Velocity	22	29	31	20	4	0

Estimate approximately the distance covered in 18 minutes by Simpson's $\frac{3}{8}$ rule.

ii) Compute $\int_0^{\frac{\pi}{2}} \sqrt{\sin x}\,dx$ using Simpson's $\frac{1}{3}$ rule taking $h = \frac{\pi}{2}$. **[2.5]**

OR

Q - 3 (a) The following values of the function $f(x)$ are given as $f(1) = 4, f(2) = 5, f(7) = 5, f(8) = 4$. Find the value of $f(6)$ and also the value of x for which $f(x)$ is maximum or minimum. **[05]**

Q - 3 (b) Apply Runge-Kutta method of fourth order to find an approximate value of y at $x = 0.6$, $\frac{dy}{dx} = \sqrt{x+y}$, $y(0.4) = 0.41$ in two step. **[05]**

Q - 4 Attempt any One. **[05]**

(i) Find the real positive root of the equation $x \sin x + \cos x = 0$, which is near $x = \pi$ correct up to four significant digits.

(ii) Calculate all the solution of the system,

$$x^2 + y^2 = 1.12, \quad xy = 0.23$$

Correct up to two decimal places.

SECTION – II

Q - 1 Do as directed. (Any Five) **[05]**

(i) Consider a sample data with values of 10,20,21,17,16 and 12.Compute the mean and median.

(ii) Consider a sample data with values of 10,20,12,17 and 16.Compute the range of the data.

(iii) Explain Qualitative data with example.

(iv) Write equation of regression line of y on x.

(v) State the central limit theorem.

(vi) Let A and B be two events with $P(A) = 0.50, P(B) = 0.60$ and $P(A \cap B) = 0.40$.Evaluate $P(A|B)$. Are A and B independent events?

(vii) A bag contains 7 white,6 red and 5 black balls. Two balls are drawn at random. Find the probability that they will both be white?

Q - 2 (a) There are two therapies B_1 and B_2 available for a curing a patient suffering from a certain disease. The patient can choose any one of the two therapies. If he selects therapy B_1 the probability of curing from the diseases is $\frac{7}{8}$ and if he selects therapy B_2 the probability of curing from the diseases is $\frac{9}{10}$. **[05]**

i) What is the probability that patient is cured from disease?

ii) Given that patient is cured, what is the probability that he has selected therapy B_2?

Q - 2 (b) The probability distribution of a random variable X is as follows **[05]**

X=x	-1	0	1	2	3
$f(x)$	0.2	0.1	k	$2k$	0.1

i) Find the value of k.

ii)Calculate the mean, variance and standard deviation.

iii)Find E$(3x+2)$ and V$(3x+2)$.

OR

Q - 2 (a) If a Poisson distribution is such that $\frac{3}{2}P(X=1) = P(X=3)$, Find **[05]**

$(i)P(X \geq 1), (ii)P(X \leq 3)$ and $(iii)P(2 \leq x \leq 5)$.

Q - 2 (b) If X is a normal variate with a mean of 120 and a standard deviation of 10, in both cases find c such that *i*) $P(X > c) = 0.02$ *ii*) $P(X < c) = 0.05$. **[05]**

Q - 3 (a) The Employment and training administration reported that the U.S. mean unemployment insurance benefit was Rs.238 per week (The world Almanac, 2003). A researcher in the state of Florida anticipated that the sample data would show evidence that the mean weekly unemployment insurance benefit in Florida was below national average. **[05]**

i) Develop appropriate such that the rejection of null hypothesis wiil support the researcher's contention.

ii) For a sample of 100 individuals, tha same mean weekly unemployment insurance benefit was Rs.231 with a sample standard deviation of Rs.80. Using 0.05 level of

significance, what is your conclusion? $(0.10 < p < 0.20)$

Q - 3 (b) From the data given below about the treatment of 250 patients suffering from a disease state whether the new treatment is superior to the conventional treatment. $(\chi_\alpha{}^2 = 3.841)$ **[05]**

Treatment	No.of Patients		Total
	Favorable	Not favorable	
New	140	30	170
Conventional	60	20	80
Total	200	50	250

OR

Q - 3 (a) Two random samples of sizes 9 and 6 gave the following value of the variable. **[05]**

Sample1	15	22	28	26	18	17	29	21	24
Sample2	8	12	9	16	15	10			

Test the difference of the estimates of the population variances at 5% level of significance.

Degree of freedom	(5,8)	(8,5)	(8,8)
5% value of F	3.69	4.82	3.44

Q - 3 (b) In a study entitled how undergraduate students use credit cards, it was reported that undergraduate students have a mean credit card balance of Rs. 3173 (Salie Mae, April 2009). This figure was an all time high and had increased 44% over the previous five years. Assume that a current study is being conducted to determine if it can be concluded that the mean credit card balance for undergraduate students has continued to increase compared to the April 2009 report. Based on previous studies. Note that population standard deviation is Rs.1000. **[05]**

i) State null and alternate hypothesis.

ii) What is the $p-$ value for a sample of 180 undergraduate students with a sample mean credit card balance of Rs. 3325?

iii) Using 0.05 level of significance, what is your conclusion?

Q - 4 Attempt any One.

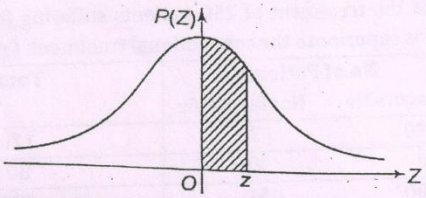
i) Calculate deviations and squared deviations about the mean for the standing salary sample data. Also find the sample variance, standard deviation and coefficient of Variation **[05]**

3450, 3550, 3650, 3480, 3355, 3310, 3490, 3730, 3540, 3925, 3520, 3480.

ii) Calculate coefficient of correlation and obtain regression line of y on x for the following data and estimate y corresponding to $x = 0.6$

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

Standard Normal (Z) Table, Area between 0 and z



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3990	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4115	0.4131	0.4147	0.4162
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

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